Quadratic Equations

Question1

If the equation $x^2 - 3ax + a^2 - 2a - k = 0$ has different real roots for every rational number a, then k lies in the interval

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Options:

A.

0

B.

4/5

C.

$\left(\frac{4}{5}, \infty\right)$

D.

None of these

Answer: C

Solution:

Given equation is

$x^2 - 3ax + a^2 - 2a - k = 0$

For distinct real roots, $D > 0$

$= (-3a)^2 - 4 \times 1 \times (a^2 - 2a - k) > 0$
$= 9a^2 - 4a^2 + 8a + 4k > 0$
$= 5a^2 + 8a + 4k > 0$

Since, leading coefficient of a^2 is $5 > 0$

So, $(8)^2 - 4 \times (5)(4k) < 0 = 64 - 80k < 0$

$\Rightarrow \quad 64 < 80k$

$$= k > \frac{4}{5} \Rightarrow k \in \left(\frac{4}{5}, \infty\right)$$

Question2

The number of all common roots of the equation $x^4 - 10x^3 + 37x^2 - 60x + 36 = 0$ and the transformed equation of it obtained by increasing any two distinct roots of it by 1 , keeping the other two roots fixed, is

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Options:

A.

1

B.

3

C.

4

D.

2

Answer: B

Solution:

Let

$f(x)=x^4-10x^3+37x^2-60x+36=0$
At $x=1, f(1)=1-10+37-60+36 \neq 0$
At $x=2, f(2)=\left(2^4-10\left(2^3+37\left(2^2\right.\right.\right.$
$-60(2+36=0$
$=16-80+148-120+36$
$=200-200=0$

So, $x=2$ is a factor of $f(x)$

Similarly $x=3$ is a factor of $f(x)$

$\therefore f(x)=(x-2)^2(x-3)^2$

$\therefore$ Original roots are $2,2,3,3$

The distinct roots are 2 and 3

There are three possible pairs of distinct roots

(i) Increase the two $2s$ by 1

(ii) Increase the two $3s$ by 1

(iii) Increase one 2 and one 3 by 1

Let us assume we choose one root 2 and one root 3 to increase by 1

Since, these distinct roots.

Increase one ' 2 ' by $1: 2+1=3$

Increase one ' 3 ' by $1: 3+1=4$

Other two roots (2 and 3) are kept fixed

$\therefore$ New sets of roots of he transformed equation is $3,4,2,3$

Let the transformed equation be $g(x)$. its roots are $2,3,3,4$

$$\therefore g(x)=(x-2)(x-3)(x-3)(x-4)$$
$$=(x-2)(x-3)^2(x-4)$$

The common roots are value of x for which $f(x)=0$ and $g(x)=0$

$\therefore$ Common roots are $2,3,3$

$\therefore$ Number of common roots $=3$

Question3

If α, β, γ are the roots of the equation $x^3-Px^2+Qx-R=0$ and $(\alpha-2)^2,(\beta-2)^2,(\gamma-2)^2$ are the roots of the equation $x^3-5x^2+4x=0$, then the possible least value of $P+Q+R$ is

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Options:

A.

5

B.

-7

C.

-1

D.

1

Answer: A

Solution:

Given, α, β, γ are the roots of the equations $x^3 - Px^2 + Qx - R = 0$

$$\therefore \quad \begin{aligned} \alpha + \beta + \gamma &= P \\ \alpha\beta + \beta\gamma + \gamma\alpha &= Q \end{aligned}$$

And $\alpha\beta\gamma = R$

Also, $(\alpha - 2)^2, (\beta - 2)^2, (\gamma - 2)^2$ are the roots of equations, $x^3 - 5x^2 + 4x = 0$

Now, $x^2 - 5x^2 + 4x = 0$

$$\begin{aligned} \Rightarrow \quad & x\left(x^2 - 5x + 4\right) = 0 \\ \Rightarrow \quad & x(x-1)(x-4) = 0 \\ \Rightarrow \quad & x = 0, 1, 4 \\ \therefore \quad & (\alpha - 2)^2 = 0 \Rightarrow \alpha = 2 \\ & (\beta - 2)^2 = 1 \Rightarrow \beta - 2 = \pm 1 \Rightarrow \beta = 1, 3 \end{aligned}$$

And $(\gamma - 2)^2 = 4 \Rightarrow \gamma - 2 = \pm 2 \Rightarrow \gamma = 0, 4$

Now,

$P + Q + R = (\alpha + \beta + \gamma) + (\alpha\beta + \beta\gamma + \gamma\alpha) + \alpha\beta\gamma$

Case I when $\alpha = 2, \beta = 1, \gamma = 0$

then,

$$
\begin{aligned}
P+Q+R & =(2+1+0)+(2+0+0) \\
& =5
\end{aligned}
$$

Case II when $\alpha=2, \beta=1, \gamma=4$

then,

$$
P+Q+R=(2+1+4)+(2+4+8)+(2 \times 1 \times 4)=29
$$

Case III when $\alpha=2, \beta=3, \gamma=0$ then

$$
\begin{aligned}
P+Q+R & =(2+3+0)+(6+0+0)+0 \\
& =11
\end{aligned}
$$

Case IV when $\alpha=2, \beta=3, \gamma=4$

then

$$
\begin{aligned}
P+Q+R & =(2+3+4)+(6+12+8)+24 \\
& =59
\end{aligned}
$$

Clearly, least value of $P+Q+R$ is 5 .

Question4

The number of integral values of ' a ' for which the quadratic equation $a x^2+a x+5=0$ cannot have real roots is

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Options:

A.

infinite

B.

20

C.

19

D.

5

Answer: C

Solution:

Given, quadratic equation is

$ax^2 + ax + 5 = 0$

For a quadratic equation

$Ax^2 + Bx + C = 0$ to have no real roots,

$$\Delta = B^2 - 4AC < 0$$

So, $\quad \Delta = a^2 - 4(a)(5) < 0$

$\Rightarrow \quad a^2 - 20a < 0$

$\Rightarrow \quad a(a - 20) < 0$

So, $\quad 0

Thus, a must be an integer.

So, the integral values of a in this range are $1, 2, 3, \ldots, 19$.

The number of such value is

$19 - 1 + 1 = 19$

Question5

If the roots of the equation $32x^3 - 48x^2 + 22x - 3 = 0$ are in arithmetic progression, then the square of the common difference of the roots is

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Options:

A.

$\frac{1}{4}$

B.

$\frac{1}{16}$

C.

$\frac{1}{9}$

D.

$\frac{1}{25}$

Answer: B

Solution:

Given equation is

$32x^3 - 48x^2 + 22x - 3 = 0$

Since, the roots of this equation are in AP .

So, sum of roots $= a - d + a + a + d = 3a$

$= \frac{-(-48)}{32} = \frac{3}{2} \Rightarrow a = \frac{1}{2}$

Now, sum of the products of roots $= \frac{22}{32}$

$\Rightarrow (a - d)a + a(a + d) + (a - d)(a + d) = \frac{11}{16}$

$$\Rightarrow a^2 - ad + a^2 + ad + a^2 - d^2 = \frac{11}{16}$$
$$\Rightarrow 3a^2 - d^2 = \frac{11}{16} \Rightarrow 3\left(\frac{1}{2}\right)^2 - d^2 = \frac{11}{16}$$
$$\Rightarrow d^2 = \frac{3}{4} - \frac{11}{16} = \frac{12 - 11}{16} = \frac{1}{16}$$
$$\Rightarrow d^2 = \frac{1}{16}$$

So, the square of the common difference of the root is $\frac{1}{16}$.

Question6

If the sum of two roots of the equation $x^4 - 2x^3 + x^2 + 4x - 6 = 0$ is zero, then the sum of the squares of the other two roots is

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Options:

A.

-6

B.

1

C.

-2

D.

0

Answer: C

Solution:

Given, equation is

$x^4 - 2x^3 + x^2 + 4x - 6 = 0$

Let the roots of this equation be α, β, γ and δ.

Sum of the roots

$= \alpha + \beta + \gamma + \delta = \frac{-(-2)}{1} = 2 \qquad \ldots (i)$

Sum of the products of roots

$$= \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$$
$$= \frac{1}{1} = 1$$

Sum of products of roots taken three at a time

$$= \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$$
$$= \frac{-4}{1} = -4$$

Product of roots $= \alpha\beta\gamma\delta = \frac{-6}{1} = -6$

Since, the sum of two roots be zero, i.e.,

$$\alpha + \beta = 0$$
$$\Rightarrow \quad \beta = -\alpha$$

So, $\gamma + \delta = 2$ [from Eq. (i)]

And,

$$\alpha(-\alpha) + \alpha\gamma + \alpha\delta + (-\alpha\gamma) + (-\alpha)\delta + \gamma\delta = 1$$
$$\Rightarrow \quad -\alpha^2 + \alpha\gamma + \alpha\delta - \alpha\gamma - \alpha\delta + \gamma\delta = 1$$
$$\Rightarrow \quad -\alpha^2 + \gamma\delta = 1 \qquad \ldots (ii)$$

And $\alpha(-\alpha)\gamma\delta = -6$

$$\Rightarrow \quad -\alpha^2\gamma\delta = -6 \qquad \ldots(iii)$$

From Eqs. (ii) and (iii), we get

$$\begin{aligned} & \gamma\delta = 1 + \alpha^2 \\ \Rightarrow \quad & \alpha^2\left(1+\alpha^2\right) = 6 \\ \Rightarrow \quad & \alpha^4 + \alpha^2 = 6 \end{aligned}$$

Let $y = \alpha^2$, then we get

$$\begin{aligned} & y^2 + y - 6 = 0 \\ \Rightarrow \quad & (y+3)(y-2) = 0 \\ \Rightarrow \quad & y = -3 \text{ and } y = 2 \end{aligned}$$

Since, α^2 must be non-negative,

$y = \alpha^2 = 2$

Then, $\gamma\delta = 1 + \alpha^2 = 1 + 2 = 3$

Now, $(\gamma+\delta)^2 = \gamma^2 + \delta^2 + 2\gamma\delta$

$$\begin{aligned} \Rightarrow \quad \gamma^2 + \delta^2 & = (\gamma+\delta)^2 - 2\gamma\delta \\ & = 2^2 - 2(3) = 4 - 6 = -2 \end{aligned}$$

Question7

If $f(x)$ is a quadratic function such that $f(x) f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$, then $\sqrt{f\left(\frac{2}{3}\right) + f\left(\frac{3}{2}\right)} =$

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. $\frac{25}{12}$

B. $\frac{10}{3}$

C. $\frac{13}{6}$

D. $\frac{41}{20}$

Answer: C

Solution:

$f(x)=1+x^2$ is the polynomial satisfying the given condition

$$f\left(\frac{2}{3}\right)=1+\left(\frac{2}{3}\right)^2=1+\frac{4}{9}=\frac{13}{9}$$

$$f\left(\frac{3}{2}\right)=1+\left(\frac{3}{2}\right)^2=1+\frac{9}{4}=\frac{13}{4}$$

$$f\left(\frac{2}{3}\right)+f\left(\frac{3}{2}\right)=\frac{13}{9}+\frac{13}{4}=13\left(\frac{1}{9}+\frac{1}{4}\right)$$

$$=\frac{13\cdot 13}{36}=\left(\frac{13}{6}\right)^2$$

$$\therefore \quad \sqrt{f\left(\frac{2}{3}\right)+f\left(\frac{3}{2}\right)}=\sqrt{\left(\frac{13}{6}\right)^2}=\frac{13}{6}$$

Question8

If α is a root of the equation $x^2-x+1=0$, then $\left(\alpha+\frac{1}{\alpha}\right)^3+\left(\alpha^2+\frac{1}{\alpha^2}\right)^3+\left(\alpha^3+\frac{1}{\alpha^3}\right)^3+\left(\alpha^4+\frac{1}{\alpha^4}\right)^3=$

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. 0

B. 1

C. -3

D. -9

Answer: D

Solution:

We have $\alpha^2-\alpha+1=0$

$\Rightarrow \quad \alpha+\frac{1}{\alpha}=1 \Rightarrow \alpha^2+\frac{1}{\alpha^2}+2=1$

$\Rightarrow \quad \alpha^2+\frac{1}{\alpha^2}=-1 \Rightarrow \alpha^4+\frac{1}{\alpha^4}+2=1$

$\Rightarrow \quad \alpha^4+\frac{1}{\alpha^4}=-1 \Rightarrow \left(\alpha+\frac{1}{\alpha}\right)^3=1$

$\Rightarrow \quad \alpha^3+\frac{1}{\alpha^3}+3\left(\alpha+\frac{1}{\alpha}\right)=1$

$\Rightarrow \quad \alpha^3+\frac{1}{\alpha^3}=-2$

$\Rightarrow \quad 1^3+(-1)^3+(-2)^3+(-1)^3$

$\Rightarrow \quad 1-1-8-1=-9$

Question9

α, β are the real roots of the equation $x^2+ax+b=0$. If $\alpha+\beta=\frac{1}{2}$ and $\alpha^3+\beta^3=\frac{37}{8}$, then $a-\frac{1}{b}=$

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. $\frac{-1}{6}$

B. $\frac{3}{2}$

C. $\frac{-3}{2}$

D. $\frac{1}{6}$

Answer: A

Solution:

Given the quadratic equation $x^2+ax+b=0$, we know from the problem statement:

The sum of the roots, $\alpha+\beta$, is $\frac{1}{2}$.

The sum of the cubes of the roots, $\alpha^3+\beta^3$, is $\frac{37}{8}$.

From Vieta's formulas:

$\alpha + \beta = -a = \frac{1}{2}$. Thus, $a = -\frac{1}{2}$.

The product of the roots $\alpha\beta = b$.

We are given that $\alpha^3 + \beta^3 = \frac{37}{8}$. Using the identity for the sum of cubes:

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

Substituting the known values:

$\alpha^3 + \beta^3 = \left(\frac{1}{2}\right)^3 - 3b\left(\frac{1}{2}\right)$

Substitute the given value for $\alpha^3 + \beta^3$:

$\frac{37}{8} = \frac{1}{8} - \frac{3b}{2}$

Rearranging gives us:

$\frac{3b}{2} = \frac{1}{8} - \frac{37}{8} = \frac{-36}{8} = \frac{-9}{2}$

Thus,

$3b = -9 \quad \Rightarrow \quad b = -3$

To find $a - \frac{1}{b}$:

$a - \frac{1}{b} = -\frac{1}{2} - \frac{1}{-3} = -\frac{1}{2} + \frac{1}{3}$

Calculate the above expression:

$a - \frac{1}{b} = \frac{-3}{6} + \frac{2}{6} = \frac{-1}{6}$

Question10

If α, β, γ are the roots of the equation $4x^3 - 3x^2 + 2x - 1 = 0$, then $\alpha^3 + \beta^3 + \gamma^3 =$

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. $\frac{2}{27}$

B. $\frac{1}{8}$

C. $\frac{3}{64}$

D. $\frac{27}{128}$

Answer: C

Solution:

$4x^3 - 3x^2 + 2x - 1 = 0 \quad$ [given]

$\alpha + \beta + \gamma = \frac{3}{4}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{2}$

and $\quad \alpha\beta\gamma = \frac{1}{4}$

We know that

$\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma$

$= (\alpha + \beta + \gamma)\left(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha\right)$

$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 - \frac{3}{4} = \frac{3}{4}\left(-\frac{7}{16} - \frac{1}{2}\right).$

$\left[\because (\alpha + \beta + \gamma)^2 = \left(\alpha^2 + \beta^2 + \gamma^2\right) + 2(\alpha\beta + \beta\gamma + \gamma\alpha)\right]$

$\left(\frac{3}{4}\right)^2 = \left(\alpha^2 + \beta^2 + \gamma^2\right) + 2 \cdot \frac{1}{2}$

$\alpha^2 + \beta^2 + \gamma^2 = \frac{9}{16} - 1 = -\frac{7}{16}]$

$$\Rightarrow \quad \alpha^3 + \beta^3 + \gamma^3 = -\frac{3}{4}\left(\frac{7+8}{16}\right) + \frac{3}{4}$$

$$= -\frac{45}{64} + \frac{3}{4} = \frac{-45+48}{64}$$

$$\alpha^3 + \beta^3 + \gamma^3 = \frac{3}{64}$$

Question11

The equation $16x^4 + 16x^3 - 4x - 1 = 0$ has a multiple root. If $\alpha, \beta, \gamma, \delta$ are the roots of this equation, then $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4} + \frac{1}{\delta^4} =$

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. $\frac{1}{64}$

B. $\frac{1}{32}$

C. 32

D. 64

Answer: D

Solution:

$16x^4 + 16x^3 - 4x - 1 = 0$

$\Rightarrow \quad (16x^4 - 1) + (16x^3 - 4x) = 0$

$\Rightarrow \quad (4x^2 + 1)(4x^2 - 1) + 4x(4x^2 - 1) = 0$

$\Rightarrow \quad (4x^2 - 1)(4x^2 + 1 + 4x) = 0$

$\Rightarrow \quad (4x^2 - 1)(2x + 1)^2 = 0 \Rightarrow x = \pm\frac{1}{2}, -\frac{1}{2}$

$\therefore \alpha = \frac{1}{2}, \beta = -\frac{1}{2}, \gamma = -\frac{1}{2}$ and $\delta = -\frac{1}{2}$

$\frac{1}{\alpha} = 2, \frac{1}{\beta} = -2, \frac{1}{\gamma} = -2$ and $\frac{1}{\delta} = -2$

$\therefore \frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^2} + \frac{1}{\delta^4}$

$= 2^4 + (-2)^4 + (-2)^4 + (-2)^4$

$= 4 \times 16 = 64$

Question12

The solution set of the equation $3^x + 3^{1-x} - 4 < 0$ contained in R is

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $(1, 2)$

B. $(1, 3)$

C. $(0, 2)$

D. $(0, 1)$

Answer: D

Solution:

To find the solution set of the inequality $3^x + 3^{1-x} - 4 < 0$, we begin by rewriting it as:

$3^x + \frac{3}{3^x} - 4 < 0$

Let $3^x = t$. Since 3^x is always positive, we have $t > 0$.

Substitute into the equation:

$t + \frac{3}{t} - 4 < 0$

Multiply the entire inequality by t (noting that $t > 0$ so the inequality sign does not change):

$t^2 - 4t + 3 < 0$

Factoring the quadratic, we get:

$(t - 3)(t - 1) < 0$

The solution to this inequality is:

$t \in (1, 3)$

Since $t = 3^x$, substitute back:

$3^x \in (1, 3)$

Taking the logarithm with base 3 of both sides gives:

$x \in (\log_3 1, \log_3 3)$

Since $\log_3 1 = 0$ and $\log_3 3 = 1$, we find:

$x \in (0, 1)$

Question13

The common solution set of the inequations $x^2 - 4x \leq 12$ and $x^2 - 2x \geq 15$ taken together is

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $(5, 6)$

B. $[5, 6]$

C. $[-3, 5]$

D. $(-\infty, -3] \cup [5, \infty)$

Answer: B

Solution:

To find the common solution set for the inequalities $x^2 - 4x \leq 12$ and $x^2 - 2x \geq 15$, we'll begin by rewriting them in a standard form:

$x^2 - 4x - 12 \leq 0$

$x^2 - 2x - 15 \geq 0$

Next, we factor each equation:

$(x - 6)(x + 2) \leq 0$

$(x - 5)(x + 3) \geq 0$

Now, we determine the solution sets for each inequality:

For $(x - 6)(x + 2) \leq 0$, the interval is $x \in [-2, 6]$.

For $(x - 5)(x + 3) \geq 0$, the intervals are $x \in (-\infty, -3] \cup [5, \infty)$.

To find the common solution set, we identify where these intervals overlap:

The intersection of $[-2, 6]$ and $(-\infty, -3] \cup [5, \infty)$ is $x \in [5, 6]$.

Thus, the common solution set for the given inequalities is $x \in [5, 6]$.

Question14

With respect to the roots of the equation $3x^3 + bx^2 + bx + 3 = 0$, match the items of List I with those fo List II

List I	List II
A All the roots are negative.	I. $(b - 3)^2 = 36 + P^2$ for $P \in R$
B Two roots are complex.	II. $-3 < b < 9$
C Two roots are positive.	III. $b \in (-\infty, -3) \cup (9, \infty)$
D All roots are real and	IV. $b = 9$
	V. $b = -3$

Options:

A. A - V, B - III, C - I, D- II

B. A - IV, B - I, C - II, D- III

C. A - V, B - II, C - III, D-I

D. A - IV, B - II, C - V, D- III

Answer: D

Solution:

Let $f(x)=3x^3+bx^2+bx+3$

(A) All the roots are negative. So, the number of sign change in $f(-x)$ should be 3 , which is possible only when b is positive. According to options (A) $\rightarrow$ IV.

(B) Clearly for any value of b, $f(x)=0$ has root $x=-1$.

$\therefore f(x)=0 \Rightarrow (x+1)\left(3x^2+(b-3)x+3\right)=0$
$\Rightarrow 3x^2+(b-3)x+3 \neq 0$

[$\because$ two roots are complex]

$\Rightarrow \quad (b-3)^2-4(3)(3)<0$

$\Rightarrow \quad b^2-6b-27<0$

$\Rightarrow \quad (b+3)(b-9)<0$

$\Rightarrow \quad -3<b<9$

Hence, B $\rightarrow$ II.

(C) We know that one root is $x=-1$

(Negative).

Two roots are complex if $-3<b<9$.

If $b=-3$, then $f(x)=0$

$\Rightarrow \quad (x+1)(x-1)^2=0$

In this case two roots are positive and equal.

(D) All roots will be real and distinct if and only if $3x^2+(b-3)x+3=0$ have two distinct real solutions other, than -1 .

$$(b-3)^2-4(3)(3)>0$$
$$\Rightarrow \quad b^2-6b-27>0$$
$$\Rightarrow \quad (b+3)(b-9)>0$$
$$\Rightarrow \quad b \in(-\infty,-3) \cup(9, \infty)$$

Thus, $\mathrm{D} \rightarrow \mathrm{III}$

Therefore, $\mathrm{A} \rightarrow \mathrm{IV}, \mathrm{B} \rightarrow \mathrm{II}, \mathrm{C} \rightarrow \mathrm{V}, \mathrm{D} \rightarrow \mathrm{III}$

Question15

If α, β are the roots of the equation $x+\frac{4}{x}=2 \sqrt{3}$, then $\frac{2}{\sqrt{3}}\left|\alpha^{2024}-\beta^{2024}\right|=$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. 2^{2024}

B. 2^{2025}

C. 2^{2023}

D. 2^{1012}

Answer: B

Solution:

Given, $x+\frac{4}{x}=2 \sqrt{3}$

$x^2-2 \sqrt{3} x+4=0$

Roots, $x=\sqrt{3} \pm i \Rightarrow \alpha, \beta=\sqrt{3} \pm i$

$$\alpha^{2024}, \beta^{2024} = (\sqrt{3} \pm i)^{2024}$$

$$= 2^{2024}\left[\frac{\sqrt{3}}{2} \pm \frac{1}{2} i\right]^{2024}$$

$$= 2^{2024}\left[\cos\frac{\pi}{6} \pm i \sin\frac{\pi}{6}\right]^{2024}$$

$$= 2^{2024}\left[\cos\frac{2024\pi}{6} \pm i \sin\frac{2024\pi}{6}\right]$$

$$\therefore \left|\, \alpha^{2024} - \beta^{2024}\right| = 2^{2024} \times \left|2i \sin\left(\frac{2024\pi}{6}\right)\right|$$

$$= 2^{2024} \times 2 \times \left|\sin\left(337\pi + \frac{\pi}{3}\right)\right|$$

$$= 2^{2025} \times \frac{\sqrt{3}}{2}$$

Hence, $\frac{2}{\sqrt{3}}\left|\alpha^{2024} - \beta^{2024}\right| = 2^{2025}$

Question16

α, β are the real roots of the equation $12x^{\frac{1}{3}} - 25x^{\frac{1}{6}} + 12 = 0$. If $\alpha > \beta$, then $6\sqrt{\frac{\alpha}{\beta}} =$

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Options:

A. $\frac{3}{2}$

B. $\frac{4}{3}$

C. $\frac{9}{8}$

D. $\frac{16}{9}$

Answer: D

Solution:

Given, $12x^{1/3} - 25x^{1/6} + 12 = 0$

Let $y = x^{1/6}$, then $y^2 = x^{1/3}$

So, $12y^2 - 25y + 12 = 0$

$$y = \frac{4}{3} \text{ or } \frac{3}{4}$$

$$x^{1/6} = \frac{4}{3} \text{ or } \frac{3}{4}$$

$$x = \left(\frac{4}{3}\right)^6 \text{ or } \left(\frac{3}{4}\right)^6$$

Here, $\alpha = \left(\frac{4}{3}\right)^6$ and $\beta = \left(\frac{3}{4}\right)^6$

$\sqrt[6]{\frac{\alpha}{\beta}} = \left(\frac{4}{3} \times \frac{4}{3}\right)^{6 \times \frac{1}{6}} = \frac{16}{9}$

Question17

α, β and γ are the roots of the equation $x^3 + 3x^2 - 10x - 24 = 0$. If $\alpha > \beta > \gamma$ and $\alpha^3 + 3\beta^2 - 10\gamma - 24 = 11k$, then $k =$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. 1

B. 11

C. 5

D. 55

Answer: C

Solution:

Given, $x^3 + 3x^2 - 10x - 24 = 0$

Factorise

$\Rightarrow (x+4)(x+2)(x-3) = 0$

Here, $\alpha = 3, \beta = -2\gamma = -4$

$\alpha^3 + 3\beta^2 - 10\gamma - 24 = 11K$

$27 + 3 \times 4 - 10 \times (-4) - 24 = 11K$

$27 + 12 + 40 - 24 = 11K \Rightarrow K = \frac{55}{11} = 5$

Question18

α, β and γ are the roots of the equation $8x^3 - 42x^2 + 63x - 27 = 0$. If $\beta < \gamma < \alpha$ and β, γ and α are in geometric progression, then the extreme value of the expression $\gamma x^2 + 4\beta x + \alpha$ is

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. $\frac{3}{4}$

B. 3

C. $\frac{3}{2}$

D. $\frac{21}{4}$

Answer: C

Solution:

Given,

$8x^3 - 42x^2 + 63x - 27 = 0$

Factorise $(4x - 3)(2x - 3)(x - 3) = 0$

Roots are $x = \frac{3}{4}, \frac{3}{2}$ or 3

So, $\beta = \frac{3}{4}, \gamma = \frac{3}{2}$ and $\alpha = 3$

Let $f(x) = \frac{3}{2}x^2 + 3x + 3$

Extreme value $= f\left(\frac{-b}{2a}\right)$

$\frac{-b}{2a} = \frac{-3}{2 \times \frac{3}{2}} \Rightarrow -1 \Rightarrow f(-1) = \frac{3}{2} - 3 + 3 = \frac{3}{2}$

Question19

If $\frac{2x^3+1}{2x^2-x-6} = ax + b + \frac{A}{Px-2} + \frac{B}{2x+q}$, then 51 apB $=$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. 23 bqA

B. 69 bqA

C. 7 bqA

D. 17 bqA

Answer: A

Solution:

Given, $\frac{2x^3+1}{2x^2-x-6}$

$2x^2-x-6)\overline{2x^3 \qquad .+1}(x+\frac{1}{2}$

$2x^3-x^2-6x$

$\frac{(-) \quad (+) \quad (+)}{x^2+6x+1}$

$x^2-\frac{1}{2}x-3$

$\frac{(-)(+) \quad (+)}{\frac{13}{2}x+4}$

$$\begin{aligned}\frac{2x^3+1}{2x^2-x-6} &= \left(x+\frac{1}{2}\right)+\frac{\frac{13}{2}x+4}{2x^2-x-6} \\ &= \left(x+\frac{1}{2}\right)+\frac{\frac{13x}{2}+4}{(2x+3)(x-2)}\end{aligned}$$

Now, $\frac{\frac{13x}{2}+4}{(2x-3)(x+2)} = \frac{A}{2x+3} + \frac{B}{x-2}$

$\frac{13x}{2}+4 = B(2x+3)+A(x-2)$

Put $x=2$

$17 = 7B \Rightarrow B = \frac{17}{7}$

Put $x = \frac{-3}{2}$

$$\frac{-39}{4} + 4 = A\left(\frac{-3}{2} - 2\right)$$

$$-\frac{23}{4} = A\left(\frac{-7}{2}\right) \Rightarrow A = \frac{23}{14}$$

$$\therefore \frac{2x^3+1}{2x^2-x-6} = \left(x + \frac{1}{2}\right)$$

$$+ \frac{23}{14(2x+3)} + \frac{17}{7(x-2)}$$

Here, $a = 1, b = \frac{1}{2}, p = 1, q = 3$

$A = \frac{17}{7}, B = \frac{23}{14}$

$\Rightarrow \quad 51\text{apB} = 51 \times 1 \times 1 \times \frac{23}{14} = \frac{1173}{14}$

$\Rightarrow 23bqA = 23 \times \frac{1}{2} \times 3 \times \frac{17}{7} = \frac{1173}{14}$

$\therefore \quad 51apB = 23bqA$

Question20

α is a root of the equation $\frac{x-1}{\sqrt{2x^2-5x+2}} = \frac{41}{60}$. If $-\frac{1}{2} < \alpha < 0$, then α is equal to

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. $-\frac{5}{31}$

B. $-\frac{7}{34}$

C. $-\frac{9}{37}$

D. $-\frac{11}{41}$

Answer: B

Solution:

Given α is a root of the equation

$$\frac{x-1}{\sqrt{2x^2-5x+2}}=\frac{41}{60}, \text{ where } -\frac{1}{2}<\alpha<0$$

$$\therefore \quad \frac{\alpha-1}{\sqrt{2\alpha^2-5\alpha+2}}=\frac{41}{60}$$

$$\Rightarrow \quad \frac{(\alpha-1)^2}{2\alpha^2-5\alpha+2}=\left(\frac{41}{60}\right)^2$$

$$\Rightarrow 3600\left(\alpha^2-2\alpha+1\right)=1681\left(2\alpha^2-5\alpha+2\right)$$

$$\Rightarrow 238\alpha^2+1205\alpha+238=0$$

$$\Rightarrow 238\alpha^2+1156\alpha+49\alpha+238=0$$

$$\Rightarrow 34\alpha(7\alpha+34)+7(7\alpha+34)=0$$

$$\Rightarrow (34\alpha+7)(7\alpha+34)=0$$

$$\Rightarrow \alpha=-\frac{7}{34},-\frac{34}{7}$$

$$\because -\frac{1}{2}<\alpha<0$$

$$\therefore \alpha=-\frac{7}{34}$$

Question21

$\alpha, \beta, \gamma, 2$ and ε are the roots of the equation

$\alpha, \beta, \gamma+4x^4-13x^3-52x^2+36x+144=0.$ If $\alpha<\beta<\gamma<2<\varepsilon$, then $\alpha+2\beta+3\gamma+5\varepsilon=$

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. -1

B. 66

C. -36

D. 48

Answer: A

Solution:

Given, roots of the equation

$x^5 + 4x^4 - 13x^3 - 52x^2 + 36x + 144 = 0$

are $\alpha_1 \beta, \gamma, 2$, and ε, where

$\alpha < \beta < \gamma < 2 < \varepsilon$

Now, $x^5 + 4x^4 - 13x^3 - 52x^2 + 36x + 144 = 0$

$\Rightarrow \quad (x+4)(x+3)(x+2)(x-2)(x-3) = 0$

$\Rightarrow \quad x = -4, -3, -2, 2, 3$

$\therefore \alpha = -4, \beta = -3, \gamma = -2$ and $\varepsilon = 3$

Now, $\alpha + 2\beta + 3\gamma + 5\varepsilon$

$= (-4) + 2(-3) + 3(-2) + 5(3)$

$= -4 - 6 - 6 + 15$

$= -16 + 15 = -1$

Question22

If the quadratic equation $3x^2 + (2k+1)x - 5k = 0$ has real and equal roots, then the value of k such that

$\frac{1}{2} < k < 0$ is

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $\frac{-16+\sqrt{255}}{2}$

B. $\frac{-16-\sqrt{255}}{2}$

C. $-\frac{2}{3}$

D. $-\frac{3}{5}$

Answer: A

Solution:

We have,

$$3x^2+(2k+1)x-5k=0$$

$\because$ roots are real and equal.

$\Rightarrow D=0$

$\Rightarrow(2k+1)^2-4(-5k)\times 3=0$

$\Rightarrow 4k^2+4k+1+60k=0$

$\Rightarrow 4k^2+64k+1=0$

$$k=\frac{-64\pm\sqrt{4096-16}}{2\times 4}$$

$$=\frac{-64\pm\sqrt{4080}}{8}=\frac{-16\pm\sqrt{255}}{2}$$

$Hence, k=\frac{-16+\sqrt{255}}{2}(\because \frac{1}{2}<k<0)$

Question23

The equations $2x^2+ax-2=0$ and $x^2+x+2a=0$ have exactly one common root. If $a\neq 0$, then one of the roots of the equation $ax^2-4x-2a=0$ is

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. 2

B. -2

C. $\frac{-4+\sqrt{22}}{3}$

D. $\frac{-2+\sqrt{22}}{3}$

Answer: D

Solution:

Let α be the common root of

$2x^2 + ax - 2 = 0$ and $x^2 + x + 0 \rightarrow 2 = 0$

$$\Rightarrow 2\alpha^2 + a\alpha - 2 = 0 \quad \ldots \text{(i)}$$

$$\alpha^2 + \alpha + 2a = 0 \quad \ldots \text{(ii)}$$

$$\Rightarrow \frac{\alpha^2}{2a^2+2} = \frac{\alpha}{-2-4a} = \frac{1}{2-a} \quad \left[a \neq 2, -\frac{1}{2}\right]$$

$$\Rightarrow \quad \alpha = \frac{4a+2}{a-2}$$

So, $\frac{(4a+2)^2}{(a-2)^2(2a^2+2)} = \frac{-1}{a-2}$

$$\frac{2(2a+1)^2}{(a^2+1)} = 2 - a$$

$$8a^2 + 2 + 8a = 2a^2 + 2 - a^3 - a$$

$$a^3 + 6a^2 + 9a = 0 \Rightarrow a\left(a^2 + 6a + 9\right) = 0$$

$$\Rightarrow a = -3 \quad [\because a \neq 0)$$

$$\therefore ax^2 - 4x - 2a = -3x^2 - 4x + 6 = 0$$

$$3x^2 + 4x - 6 = 0$$

$$x = \frac{-4 \pm \sqrt{16+72}}{6}$$

$$= \frac{-2 \pm \sqrt{22}}{3}$$

So, one root of $ax^2 - 4x - 2a = 0$ is

$= \frac{-2+\sqrt{22}}{3}$.

Question24

If α, β and γ are the roots of the equation $2x^3 - 3x^2 + 5x - 7 = 0$, then $\sum \alpha^2\beta^2 =$

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $-\frac{17}{4}$

B. $\frac{17}{4}$

C. $-\frac{13}{4}$

D. $\frac{13}{4}$

Answer: A

Solution:

We have,

$$2x^3 - 3x^2 + 5x - 7 = 0$$

$$\therefore \quad \alpha + \beta + \gamma = -\left(-\frac{3}{2}\right) = \frac{3}{2} \quad \ldots \text{(i)}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{5}{2} \quad \ldots \text{(ii)}$$

$$\alpha\beta\gamma = \frac{7}{2} \quad \ldots \text{(iii)}$$

On squaring both side of Eq. (ii), we get

$$(\alpha\beta+\beta\gamma+\gamma\alpha)^2=\frac{25}{4}$$

$$\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2+2$$

$$\left(\alpha\beta^2\gamma+\alpha\beta\gamma^2+\alpha^2\beta\gamma\right)=\frac{25}{4}$$

$$\Sigma\alpha^2\beta^2+2\alpha\beta\gamma(\alpha+\beta+\gamma)=\frac{25}{4}$$

From Eqs. (i) and (iii), we get

$$\Sigma\alpha^2\beta^2+2\left(\frac{7}{2}\right)\left(\frac{3}{2}\right)=\frac{25}{4}$$

$$\Sigma\alpha^2\beta^2=\frac{25}{4}-\frac{21}{2}=-\frac{17}{4}$$

Question25

The sum of two roots of the equation $x^4-x^3-16x^2+4x+48=0$ is zero. If α, β, γ and δ are the roots of this equation, then $\alpha^4+\beta^4+\gamma^4+\delta^4=$

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. 123

B. 369

C. 132

D. 396

Answer: B

Solution:

We have,

$x^4-x^3-16x^2+4x+48=0 \quad \ldots$ (i)

By hit and trial, we get, x^2-4 is the factor of

$$x^4 - x^3 - 16x^2 + 4x + 48 = 0$$

$$\Rightarrow \quad \frac{x^4 - x^3 - 16x^2 + 4x + 48}{x^2 - 4}$$

$$\Rightarrow x^2 - x - 12$$

$$\Rightarrow x^4 - x^3 - 16x^2 + 4x + 48 = \left(x^2 - 4\right)$$

$$\left(x^2 - x - 12\right)$$

$$\Rightarrow (x-2)(x+2)(x-4)(x+3) = 0$$

$\Rightarrow$ Roots of above Eq. (i) are,

$-3, -2, 2, 4$

So,

$$\alpha^4 + \beta^4 + \gamma^4 + \delta^4 = (-3)^4 + \left(-2^4\right.$$

$$+\left(2^4 + (4)^4 = 369\right.$$

Question26

The set of all values of x which satisfy both the inequations $x^2 - 1 \leq 0$ and $x^2 - x - 2 \geq 0$ simultaneously is

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $(-1, 2)$

B. $(-1, 1)$

C. $(-2, -1)$

D. $\{-1\}$

Answer: D

Solution:

Given that

$$x^2 - 1 \leq 0 \qquad \ldots (i)$$
$$\Rightarrow \quad x^2 \leq 1$$
$$\Rightarrow \quad -1 \leq x \leq 1$$
and also $x^2 - x - 2 \geq 0 \qquad \ldots (ii)$
$$\Rightarrow (x-2)(x+1) \geq 0$$
$$\Rightarrow x \in (-\infty, -1] \cup [2, \infty)$$

Thus, the set of all value of x which satisfy both the inequalities $x^2 - 1 \leq 0$ and $x^2 - x - 2 \geq 0$ simultaneously is $\{-1\}$.

Question27

The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. If the other roots of the first and second equations are integers and are in the ratio $4:3$, then their common root is

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. 4

B. 3

C. 2

D. 1

Answer: C

Solution:

The given equations are

$x^2 - 6x + a = 0 \qquad \ldots (i)$

$and\ x^2 - cx + 6 = 0 \qquad \ldots (ii)$

Since, α is a common roots in both the equations

Thus, the other roots of first and second equations are 4β and 3β.

Hence, $(\alpha, 4\beta)$ and $(\alpha, 3\beta)$ are the roots of first and second equations.

Thus, $4\alpha\beta = a$ and $3\alpha\beta = 6$

$\alpha\beta=\frac{a}{4}\Rightarrow a=8$

So, from Eq. (i), we get

$x^2-6x+8=0\Rightarrow x=2$ and 4

If $x=2$ is a common roots, then

From Eq. (ii), we get $4-2c+6=0$

$\Rightarrow c=5$
$\therefore x^2-5x+6=0\Rightarrow x=3,2$

Thus, their common root is 2 .

Question28

If α and β are the roots of the equation $x^2+2x+2=0$, then $\alpha^{15}+\beta^{15}=$

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. -512

B. -256

C. 256

D. 512

Answer: B

Solution:

To find $\alpha^{15}+\beta^{15}$ for the roots α and β of the equation $x^2+2x+2=0$, let's solve the quadratic equation:

$x^2+2x+2=0$

First, we complete the square:

$(x+1)^2=-1$

This implies:

$x+1=\pm i$

Thus, the roots are:

$x = i - 1 \quad \text{or} \quad x = -i - 1$

Assign $\alpha = i - 1$ and $\beta = -i - 1$.

Next, calculate $\alpha^{15} + \beta^{15}$:

$\alpha^2 = i^2 - 2i + 1 = -1 - 2i + 1 = -2i$

$\beta^2 = (-i)^2 - 2(-i) + 1 = -1 + 2i + 1 = 2i$

Now express $\alpha^{15} + \beta^{15}$ as:

$\alpha^{15} + \beta^{15} = (\alpha^2)^7 \cdot \alpha + (\beta^2)^7 \cdot \beta$

Substitute α^2 and β^2:

$= (-2i)^7 \cdot (i - 1) + (2i)^7 \cdot (-i - 1)$

Simplify:

Calculate $(-2i)^7$:

$(-2i)^7 = (-2)^7 \cdot i^7 = -128 \cdot (-i) = 128i$

Calculate $(2i)^7$:

$(2i)^7 = 2^7 \cdot i^7 = 128 \cdot (-i) = -128i$

Substitute back:

$= 128i(i - 1) - 128i(-i - 1)$

Simplify the expression:

$= 128i \cdot i - 128i \cdot 1 - 128i \cdot (-i) - 128i \cdot (-1)$

$= 128(-1) - 128i + 128 - 128i$

Combine terms:

$= 128(-1 + 1) + 128i - 128i$

$= 256(-1) = -256$

Thus, $\alpha^{15} + \beta^{15} = -256$.

Question29

If the equation whose roots are P times the roots of the equation $x^4 - 2ax^3 + 4bx^2 + 8ax + 16 = 0$ is a reciprocal equation, then $|P| =$

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. 2

D. 3

Answer: B

Solution:

Consider the equation:

$$x^4 - 2ax^3 + 4bx^2 + 8ax + 16 = 0 \qquad (i)$$

Let the roots of equation (i) be α, β, γ, and δ.

The product of the roots for polynomial equation (i) is given by:

$$\alpha\beta\gamma\delta = \frac{\text{Constant term}}{\text{Coefficient of } x^4} = \frac{16}{1} = 16 \qquad (ii)$$

Suppose we have a new equation whose roots are P times the roots of equation (i), thus the roots become $P\alpha$, $P\beta$, $P\gamma$, and $P\delta$.

Now, the product of these new roots is:

$$P\alpha \cdot P\beta \cdot P\gamma \cdot P\delta = P^4 \times \alpha\beta\gamma\delta = 1$$

So,

$$P^4 = \frac{1}{\alpha\beta\gamma\delta}$$

Substituting the value from equation (ii), we get:

$$P^4 = \frac{1}{16}$$

Thus,

$$|P| = \frac{1}{(16)^{\frac{1}{4}}} = \frac{1}{2}$$

Therefore, the absolute value of P is $\frac{1}{2}$.

Question30

If one root of the equation $4x^2 - 2x + k - 4 = 0$ is the reciprocal of the other, then the value of k is

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. -8

B. 8

C. -4

D. 4

Answer: B

Solution:

Given the quadratic equation $4x^2 - 2x + (k-4) = 0$, one of the roots is the reciprocal of the other. To find the value of k, we first compare the equation to the standard form:

$ax^2 + bx + c = 0$

Here, we identify:

$a = 4, \quad b = -2, \quad c = k - 4$

Since one root is the reciprocal of the other, let α and β be the roots. The relationship between the roots is:

$\alpha = \frac{1}{\beta} \quad \Rightarrow \quad \alpha\beta = 1$

The product of the roots of a quadratic equation is given by:

$\alpha \cdot \beta = \frac{c}{a}$

Substituting the known values:

$1 = \frac{k-4}{4}$

To solve for k, we set:

$k - 4 = 4$

Thus, solving for k, we find:

$k = 4 + 4 = 8$

Therefore, the value of k is 8.

Question31

Two numbers b and c are chosen at random in succession without replacement from the set $\{1, 2, 3, \ldots\ldots, 9\}$. Then, the probability that $x^2 + bx + c > 0, \forall x \in R$ i

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. $\frac{29}{72}$

B. $\frac{32}{81}$

C. $\frac{45}{143}$

D. $\frac{82}{125}$

Answer: A

Solution:

$n(S) = {}^9C_1 \times {}^8C_1 = 72$
$x^2 + bx + c > 0 \Rightarrow b^2 - 4c < 0$

b	c	Number of cases
1	2,3,4,...9	8
2	3,4,5,...9	7
3	4,5,6,...9	6
4	5,6,7,8,9	5
5	7,8,9,	3
6	ϕ	0
		Total =29

$\therefore n(E) = 29$

$\therefore$ Required probability $= \frac{29}{72}$

Question32

If $(x-2)$ is a common factor of the expressions x^2+ax+b and x^2+cx+d, then $\frac{b-d}{c-a}=$

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. 1

B. 2

C. 3

D. 4

Answer: B

Solution:

Given, $(x-2)$ is a common factor of equation $\left(x^2+ax+b\right)$ and

$$\left(x^2+\alpha+d\right).$$

$$\therefore x^2+ax+b=(x-2)(x-\alpha)$$
$$x^2+ax+b=x^2-(2+\alpha)x+2\alpha$$
$$\therefore 2\alpha=b; \quad \therefore \alpha=\frac{b}{2}$$
$$\text{and } -2-\alpha=a$$
$$-2-\frac{b}{2}=a$$

$$x^2+\alpha+d=(x-2)(x-\beta)$$
$$x^2+\alpha+d=x^2-(2+\beta)x+2\beta$$
$$d=2\beta$$
$$\therefore \quad \frac{d}{2}=\beta$$
$$\therefore \quad -2-\beta=c$$
$$\therefore \quad \frac{b-d}{c-a}=\frac{2\alpha-2\beta}{-2-\beta+2+\alpha}$$
$$=\frac{2(\alpha-\beta)}{(\alpha-\beta)}=2$$

Question33

The sum of the roots of the equation
$e^{4t} - 10e^{3t} + 29e^{2t} - 20e^{t} + 4 = 0$ **is**

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. $\log_e 10$

B. $2\log_e 2$

C. $\log_8 2$

D. $2\log_{810}$

Answer: B

Solution:

Given,

$$e^{4t} - 10e^{3t} + 29e^{2t} - 20e^{t} + 4 = 0$$

Let $e^t = x$

$$\Rightarrow x^4 - 10x^3 + 29x^2 - 20x + 4 = 0$$

$$\Rightarrow \quad x^2 - 10x + 29 - \frac{20}{x} + \frac{4}{x^2} = 0$$

$$\Rightarrow x^2 + \left(\frac{2}{x}\right)^2 - 10\left(x + \frac{2}{x}\right) + 29 = 0$$

$$\Rightarrow \left(x + \frac{2}{x}\right)^2 - 10\left(x + \frac{2}{x}\right) + 25 = 0$$

$= 5$

Let

$$\therefore \quad x + \frac{2}{x} = P$$

$$\therefore \quad P^2 - 10P + 25 = 0$$

$$\Rightarrow \quad (P-5)^2 = 0$$

$$\Rightarrow \quad P$$

Putting the value of P in Eq. (ii), we get

$$x + \frac{2}{x} = 5$$

$$\Rightarrow \quad x^2 - 5x + 2 = 0$$

$$\Rightarrow \quad x = \frac{5 \pm \sqrt{25 - 8}}{2}$$

$$\therefore \quad x = \frac{5 + \sqrt{17}}{2}, \frac{5 - \sqrt{17}}{2}$$

$$e^{t_1} = \frac{5 + \sqrt{17}}{2},$$

$$e^{t_2} = \frac{5 - \sqrt{17}}{2}$$

$$e^{t_1} \cdot e^{t_2} = \frac{25 - 17}{4} = \frac{8}{4} = 2$$

$$e^{t_1} + t_2 = 2$$

$$t_1 + t_2 = \log_e 2$$

$\because$ There are 4 roots and 2 of them are repeated.

$\Rightarrow t_1 + t_2 + t_3 + t_4 = 2 \log_e 2$
